

M434 STOP PRESS 1 2001

## Differential Geometry

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**1 Introduction**

Welcome to M434. This Stop Press contains important information, so please read it carefully as it contains last minute news affecting your study.

This first Stop Press is mainly concerned with important general matters to do with being an Open University student, but there is also course-specific information about the material. The general material appears in Section 2 immediately after this introduction and the course-specific items follow in Section 3.

It is suggested that you use the general material for reference. You should read the course-specific items *before* doing anything else.

Bob Margolis  
Chair, M434

## 2 General Information

### 2.1 Useful Telephone Numbers and Evening Advice Line

Sometimes students have called the Faculty with queries that would be more easily answered in other areas of the University. This has been frustrating and irritating for you, particularly if you have been redirected several times before speaking to someone who can help you. Below are some telephone numbers from various areas of the University, which you can keep for quick reference. Your Student Handbook also contains information about contacts within the University. Remember to have your PI number ready when you phone.

There is an additional evening service which aims to provide help to students who are unable to make contact with their Regional Centre during the day. The service is run by a team of student service assistants, advisors and associate lecturers between 5.00 pm and 9.00 pm Monday to Friday (excluding Bank Holidays). The telephone number is 0541 596953. Calls are charged at the national rate. Your query will be dealt with nationally but referred back to Student Services in your Regional Centre the next working day, if appropriate. This service is available to all students, but those allocated to tutor-counsellors should contact them for personal advice as usual.

Any queries which involve direct questions about the administration of the course should be addressed to the Course Manager for the course as follows: The Course Manager (M434), Faculty of Mathematics and Computing, The Open University, Walton Hall, Milton Keynes MK7 6AA or telephone +44 (0) 1908 652333.

The Course Manager may also be able to deal with other administrative queries but please try the numbers below first where appropriate. The telephone number and address of your Regional Centre is listed in your Student Handbook.

Nature of Query	Contact	Telephone No/Fax
<b>Registration problem</b> e.g. Change of course What to do about materials received for a course already declined Withdrawals	Registration & Fees Centre or Student Services section in Regional Centre	Tel: 01908-95____ followed by the four digits relevant to the first letter of your surname B D R U V 8135 C G I P Q 8130 A E H L Y 8131 F M O W Z 8120 J K N S T X 8136 Fax: 01908-654914
<b>Awards</b>	Awards & Ceremonies Centre	Tel: 01908-653003
Credit Transfer enquiries	Credit Transfer Centre	Tel: 01908-653077 Fax: 01908-654918
<b>Fees</b> e.g. Fee queries; refunds	Registration & Fees Centre or Student Services section in Regional Centre	Tel: 01908-95____ followed by the four digits relevant to the first letter of your surname B D R U V 8135 C G I P Q 8130 A E H L Y 8131 F M O W Z 8120 J K N S T X 8136 Fax: 01908-654914
Queries about fee payment by instalments	OU Students Budget Account Office	Tel: 01908-655777 Fax: 01908-654903



Nature of Query	Contact	Telephone No/Fax
<b>Mailings</b>		
Missing items in mailings or any queries about contents and timings of mailings	Milton Keynes Distribution Services. Ask for 'enquiries' and have your PI number ready.	Tel: (8.30 am to 5.00 pm) 01908-655739 other times 01908-233842 Fax: 01908-856611
<b>Assignments</b>		
TMA queries/late submission	Assignment Handling	Tel: 01908-654330
CMA queries	Assignment Records	Tel: 01908-653702 01908-655716
Please note that TMA/CMA grades will not be discussed, only queries about receipt of assignments etc. Possible errors or marks awarded/late submission of TMAs should be discussed with your tutor in the first instance.		
<b>Regional Arrangements</b>		
Tutorial/Day Schools	Contact Student Services staff in your own Regional Centre	
Tutor Allocations		
Exam Centres and special arrangements		
Course Choice and Degree/Diploma planning advice and vocational guidance		

## 2.2 Telephone Tutorial Service

Throughout the year, members of the Mathematics and Computing Faculty are available to answer questions about courses. Below is a list of names of people who have volunteered to answer questions about M434, their telephone numbers and the times when they are usually available.

Feel free to use this service, but please note the following.

1. Your tutor, if available, should normally be the first contact for a course-related query. The telephone tutorial service is a back-up facility.
2. Please do not call outside the times listed, since the people involved will not then be expecting your call and may be engaged in other activities.
3. The people on the list undertake to be available normally at the times listed, but other commitments may make them unavailable on certain occasions.
4. Please note that phone numbers for evenings and weekends are volunteers' home numbers, and for these it will usually not be appropriate to leave a message asking to be called back. Please try the number again later or try another number if one is given.
5. Although TMA questions are not excluded as sources of queries, you should expect the amount of information provided in response to such a query to be limited, in fairness to other students. (TMAs should be substantially your own work, since the grades obtained on them count towards your overall course assessment.)

When you use the telephone tutorial service, please mention first that you are calling about M434; the people involved may be answering questions about several courses. It is a good idea to have a pencil and paper handy, and any course materials that may be relevant. A text reference is usually a helpful starting point.

### M434 Putting computer systems to work

Name	Day	Time	Tel Number
Bob Margolis	Thursday	19.30–21.30	01252-871077

## 2.3 Contacting the Faculty

A message recording system is available in the Faculty of Mathematics and Computing on 01908-653243. After a few rings, you will either be able to speak with a member of staff or you will hear the following message:

"Hello, this is the Courses Office, Faculty of Mathematics and Computing. No one is available to take your call at the moment. Please leave your name telephone number, PI number together with your course code and we will get back to you. Thank you."

You should then begin recording your message at the tone and just hang up when you have finished. (You can ignore the other instructions.)

Many messages received via this service are incomplete or unclear, so please speak clearly and slowly.

You may also contact the Course Manager by e-mail at the following address.

m.e.jenkins@open.ac.uk

Finally, we regard comments and complaints as an important source of information for maintaining and improving standards. Our aim is to sort out difficulties promptly and efficiently, but if you are dissatisfied then you may wish to complain. Should you wish to do this please put your complaint in writing (not e-mail) and address it to the Course Manager for your course.

## 2.4 The University Scale and Faculty of Mathematics and Computing Courses

All aspects of your assessment for this course will be marked on the University Scale, which is detailed in the table below. Your assignment scores will be combined as indicated on your Study Calendar to give an overall continuous assessment score (which will also be quoted on this Scale). In order to ensure being awarded a particular grade of pass, you must achieve the minimum scores quoted for that grade on both overall continuous assessment and the examinable component. So, for example, to ensure that you obtain a Pass 4 on your course you will need to get a score of at least 40 for both your overall continuous assessment and your examinable component.

Band	University Scale Score	Performance Standard
A	85-100	Pass 1
B	70-84	Pass 2
C	55-69	Pass 3
D	40-54	Pass 4
E	30-39	Bare Fail
F	15-29	Fail
G	0-14	Bad Fail

It has been our experience in the Faculty of Mathematics and Computing that students usually score better on continuous assessment than in the examination. A major reason for this is that the questions set in assignments on most Mathematics and Computing courses can, in principle, be answered 100 per cent correctly using the information in the relevant course units. Of course, students rarely do answer all questions in an assignment correctly; but it is only to be expected that high scores will often be obtained and, in particular, higher scores than are usually obtained for courses in other faculties. We try to ensure that examination questions are not harder than continuous assessment questions, but, even so, it is quite a different matter to tackle similar questions under examination conditions.

You should bear in mind this discrepancy between examination and continuous assessment scores when trying to predict your final grade on the basis of your continuous assessment score. As a very rough guide, it may help you to know that the mean overall continuous assessment scores on Mathematics and Computing Faculty courses in 1999 (the latest year for which figures are available) ranged between 65 and 87 percent, while the mean examination scores ranged between 49 and 71 percent. We anticipate that similar differences will be obtained this year.

We should like to emphasise that there is no reason to be concerned about this discrepancy, which is anticipated by the Examination and Assessment Board for your course and taken into account in deciding your result status, as described in the Student Handbook.



# The M500 Society

If you are interested in mathematics, joining the M500 Society could be a way of alleviating the isolation of studying alone. It is a society for OU students, staff and friends, which through a magazine published several times a year, provides a forum for discussion, comment and argument, as well as fun! The Society also publishes a directory of members who agree to be sources of help and advice on listed courses. It runs a popular weekend each September for examination preparation, for which members are entitled to discount, as well as a weekend in January for mathematical fun.

Enquiries: please send stamped A5 envelope for membership form and free magazine

Name: Glenda Franklin

Position in society: New Members Secretary

Address: 16 Warbank Close, Alvechurch, Birmingham, B48 7PA

## 3 Course Specific Information

### 3.1 Errata

Page	Location	Correction
Part 0	page 7, line 22	Newton's year of birth should be 1642 not 1647
Part I	page 30, Solution 4.2	$\alpha_3(t) = e^t - 6$ (not $e^t - 5$ ) so $\alpha(t) = \left(\frac{t^3}{3} + 1, \frac{t^2}{2}, e^t - 6\right)$
Part I	page 31, RH column, last lines of Solution 5.2 (b), (c)	the bracketed term should be $(x^2 + xz + y^2)$ in both cases
Part II	page 19, line -20	should read $= \nabla_{\alpha'(s)} E_1$
Part III	page 13, line -10	the third term should be $(-\bar{\kappa}\bar{T} + \bar{\tau}\bar{B}) \cdot N$
Part IV	page 10, lines -14 to -12	should read "This is zero wherever $v = 0$ and so...with the $x$ -axis deleted..."
Part IV	page 11, line -5	should read $\nabla g = \sum \left(\frac{\partial g}{\partial x_i}\right) U_i$
Part IV	page 14, line 8	$\bar{u}$ and $\bar{v}$ are better described as functions $D_1 \rightarrow \mathbf{R}$
Part IV	page 29, line -10	should read $\mathbf{x}(u, v) = \dots$ (no suffix)
Part IV	page 35, RH column, line 9	should read "Hence $\mathbf{x}(\mathbf{E}^2) \subseteq M \dots$ "
Part IV	page 39, Solution 5.1, line 3	should read $\dots = \cos v \dots$
Part IV	page 39, Solution 5.1, lines 22, 29	replace $\mathbf{y}_u$ by $\mathbf{y}_v$
Part V	page 12, line -21	the matrix should be $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Part V	page 18, lines 3, 5	the expressions for $b$ and $d$ should be $b = \frac{-Fl+Em}{EG-F^2}, d = \frac{-Fm+En}{EG-F^2}$ This error recurs in the calculations that follow. The final results are correct!
Part V	page 27, RH column, line -11	should read "Since $\alpha'' \times U = 0 \dots$ "
Part VI	page 6, lines 15, 18	should read $\dots = \nabla_{E_1} E_3$ The final matrix is correct and is the transpose of what O'Neill quotes on the referenced page.
Part VI	page 8, line -18	last term should be $(f^2 + g^2)\theta_1 \wedge \theta_2$
Specimen Exam Question 5(b) Solutions to		should read $\alpha(t) = (\sin(t/\sqrt{2}), \cos(t/\sqrt{2}), t/\sqrt{2})$
Specimen Exam Question 11, pages 9-10, line 3		the $B$ term should be $v\tau B$ . This error propagates through the rest of the solution.

## 3.2 Comments

Part IV, page 27, line -16:

There is still a possible ambiguity over the sign of the unit normal. However, by selecting minus the given expression if necessary, the ambiguity will be resolved.

We usually use the technique where  $\mathbf{x}$  is a parametrization and the various patches arise by choosing a collection of domains for a fixed rule. In this case, there cannot possibly be a sign ambiguity on the overlap, since  $\mathbf{x}_u$  and  $\mathbf{x}_v$  are actually identical there.

Part IV, page 34, RH column, Solution 2.5(c):

The attack there is flawed. It is much better to observe that

$$v = \frac{z}{b}$$

and that

$$x \cos(v) - y \sin(v) = 0.$$

Combining these gives

$$x \cos\left(\frac{z}{b}\right) - y \sin\left(\frac{z}{b}\right) = 0.$$

Thus

$$g = x \cos\left(\frac{z}{b}\right) - y \sin\left(\frac{z}{b}\right), \quad c = 0.$$

This avoids all worries about zeros and is also correct.

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### Differential Geometry

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# Introduction

This Stop Press is necessary because a new edition of *O'Neill* has appeared. Unfortunately, unlike the minor revisions that have happened from time to time, this time exercises have gone or been changed, page numbers altered and so on. Usually, publishers notify of impending new editions, for some reason this did not happen; please accept my apologies that the system appears to have failed.

I feel that the minor changes in page numbers will be irritating but manageable — you are always asked to read a section at a time. The exercises are a bigger problem. To avoid missing anything, *any* exercise from the old version *O'Neill* that has been changed is usually given here in its original form. Thus, you will find that a few exercises are here, not in *O'Neill* as stated in the course texts. Only if exercises contain diagrams have you been referred to the newly numbered exercise in *O'Neill*.

Sections correspond to the course texts, subsections to the sections within texts. The exercises are numbered as they are in the course texts, *not* as in *O'Neill*. Only subsections containing changes actually appear.

There are no exercises in Part 0 taken from *O'Neill*, so there is no corresponding section.

There is a further complication. After originally making considerable play of the fact that  $\mathbf{R}^3$  is familiar 3-dimensional Euclidean space, whereas  $\mathbf{E}^3$  is the same but equipped with all possible tangent vectors at all points, *O'Neill* now plays down the difference.

It is safe to read  $\mathbf{E}^n$  and  $\mathbf{R}^n$  interchangeably if you are using the second edition of *O'Neill*. (Here  $n = 1, 2$  or  $3$ .)

I am deeply indebted to Mary Jones (M434 tutor) for her assistance in digging us out of this unfortunate situation.

Please note that there are also three corrections to TMAs, one in each of TMAs 02, 03 and 04.

Finally, my telephone number for the telephone help line has changed, it is now

01252 651001

(the day and times remain unchanged).

Bob Margolis  
Chair, M434

## 1 Part I

### Curves in $\mathbf{E}^3$

#### Exercise 4.1

Please ignore the reference to Example 4.2(3) of *O'Neill* which now uses different notation.

### Mappings

No exercises from *O'Neill* at the end of this subsection. However, the references on page 24, line 9, should be to *O'Neill*, page 40, Exercises 9(a) and 9(b).



## 2 Part II

### Curves

#### Exercise 2.4

Let  $\alpha, \beta: I \rightarrow \mathbf{E}^3$  be curves such that  $\alpha'(t)$  and  $\beta'(t)$  are parallel (same Euclidean coordinates) for each  $t$ . Prove that  $\alpha$  and  $\beta$  are *parallel*, in the sense that there exists a point  $\mathbf{p}$  such that  $\beta(t) = \alpha(t) + \mathbf{p}$  for all  $t$ .

### The Frenet Formulas

#### Exercise 3.5

*Curves in the plane.* For a unit-speed curve  $\beta(s) = (x(s), y(s))$  in  $\mathbf{E}^2$ , the *unit tangent* is  $T = \beta' = (x'(s), y'(s))$ , but the *unit normal*  $N$  is defined by rotating  $T$  through  $+90^\circ$  so that  $N = (-y', x')$ . Thus,  $T'$  and  $N$  are collinear and the *curvature* of  $\beta$  is defined by the Frenet equation  $T' = \kappa N$ . Prove

(a)  $N' = -\kappa T$ ; (b) if  $\varphi$  is the slope angle of  $\beta$  then  $\kappa = \varphi'$ .

### Arbitrary-speed Curves

#### Exercise 4.4

If  $\alpha$  is a curve with *constant speed*  $c > 0$ , show that

$$\begin{aligned}T &= \alpha'/c, \\N &= \alpha''/\|\alpha''\|, \\B &= \alpha' \times \alpha''/c\|\alpha''\|, \\\kappa &= \|\alpha''\|/c^2, \\\tau &= \frac{\alpha' \times \alpha'' \cdot \alpha'''}{c^2\|\alpha''\|^2}\end{aligned}$$

#### Exercise 4.5

Let  $\alpha$  be a cylindrical helix with unit vector  $\mathbf{u}$ , angle  $\vartheta$  and arc-length function  $s$  (measured from, say,  $t = 0$ ). The unique curve  $\gamma$  such that  $\alpha(t) = \gamma(t) + s(t) \cos \vartheta \mathbf{u}$  is called the *cross-section curve* of the cylinder on which  $\alpha$  lies. Prove that

- (a)  $\gamma$  lies in the plane through  $\alpha(0)$  orthogonal to  $\mathbf{u}$ .  
(b) The curvature of  $\gamma$  is  $\kappa/\sin^2 \vartheta$ , where  $\kappa$  is the curvature of  $\alpha$ .

(Hint: for (b) it suffices to assume that  $\alpha$  has unit speed.)

#### Exercise 4.6

If  $\beta$  is a unit-speed curve with  $\kappa > 0$  and  $\tau \neq 0$  both constant, prove that  $\beta$  is a (circular) helix.

#### Exercise 4.7

Let  $\sigma$  be the spherical image of a unit-speed curve  $\beta$ . Prove that the curvature and torsion of  $\sigma$  are

$$\begin{aligned}\kappa_\sigma &= \sqrt{1 + (\tau/\kappa)^2}, \\\tau_\sigma &= \frac{(d/ds)(\tau/\kappa)}{\kappa(1 + (\tau/\kappa)^2)},\end{aligned}$$

where  $\kappa$  and  $\tau$  are the curvature and torsion of  $\beta$ .

#### Exercise 4.8

Prove that a curve is a cylindrical helix if and only if its spherical image is part of a circle. (No computations needed.)

## Covariant Derivatives

### Exercise 5.4

If  $W = \sum w_i U_i$  is a vector field on  $\mathbf{E}^3$ , the *covariant differential* of  $W$  is defined to be  $\nabla W = \sum dw_i U_i$ . Thus,  $\nabla W$  is the function on all tangent vectors whose value on  $\mathbf{v}_p$  is

$$\sum dw_i(\mathbf{v}) U_i(\mathbf{p}) = \nabla_{\mathbf{v}} W.$$

Compute the covariant differential of

$$w = xy^3 U_1 - x^2 z^2 U_3,$$

and use it to find  $\nabla_{\mathbf{v}} W$ , where

(a)  $\mathbf{v} = (1, 0, -3)$  at  $\mathbf{p} = (-1, 2, -1)$ ,

(b)  $\mathbf{v} = (-1, 2, -1)$  at  $\mathbf{p} = (1, 3, 2)$ .

## The Structural Equations

### Exercise 8.2

For the cylindrical frame field  $E_1, E_2, E_3$ :

(a) Prove that  $\theta_1 = dr$ ,  $\theta_2 = d\vartheta$ ,  $\theta_3 = dz$  by evaluating on  $U_1, U_2$  and  $U_3$ .

(b) Deduce that  $E_1[r] = 1$ ,  $E_2[\vartheta] = 1/r$ ,  $E_3[z] = 1$  and that the other six possibilities are all zero.

(c) For a function  $f(r, \vartheta, z)$  expressed in terms of cylindrical coordinates, show that

$$E_1[f] = \frac{df}{dr}, \quad E_2[f] = \frac{1}{r} \frac{df}{d\vartheta}, \quad E_3[f] = \frac{df}{dz}.$$

## 3 Part III

## Derivative Maps of Isometries

### Exercise 2.2

Given the frame

$$\mathbf{e}_1 = (2, 2, 1)/3, \quad \mathbf{e}_2 = (-2, 1, 2)/3, \quad \mathbf{e}_3 = (1, -2, 2)/3$$

at  $\mathbf{p} = (0, 1, 0)$  and the frame

$$\mathbf{f}_1 = (1, 0, 1)/\sqrt{2}, \quad \mathbf{f}_2 = (0, 1, 0), \quad \mathbf{f}_3 = (1, 0, -1)/\sqrt{2}$$

at  $\mathbf{q} = (3, -1, 1)$ , find the isometry  $F = TC$  which carries the  $\mathbf{e}$  frame to the  $\mathbf{f}$  frame.

## Orientation

### Exercise 3.1

Find the orientations of the two frames in Exercise 2.2 above.

## Congruence of curves

### Exercise 5.4

Show that the curve

$$\beta(t) = (t + \sqrt{3} \sin t, 2 \cos t, \sqrt{3}t - \sin t)$$

is a helix by computing its curvature and torsion. Find a helix  $\alpha$  of the form

$$(a \cos t, a \sin t, bt)$$

and an isometry  $F$  such that  $F(\alpha) = \beta$ .

## 4 Part IV

### Patch Computations

#### Exercise 2.3

A cone is a ruled surface with parametrization of the form

$$\mathbf{x}(u, v) = \mathbf{p} + v\delta(u).$$

Thus, all rulings pass through the vertex  $\mathbf{p}$ . Show that the regularity of  $\mathbf{x}$  is equivalent to both  $v$  and  $\delta \times \delta'$  never zero. (Thus the vertex is never part of the cone.)

#### Exercise 2.4

A cylinder is a ruled surface with parametrization of the form

$$\mathbf{x}(u, v) = \beta(u) + v\mathbf{q}.$$

Thus the rulings are all parallel. Prove that the regularity of  $\mathbf{x}$  is equivalent to  $\beta' \times \mathbf{q}$  never zero.

#### Exercise 2.5

This is now O'Neill, exercise 5 (a)–(c).

#### Exercise 2.6

In each case, (i) prove that  $M$  is a surface and (ii) show that  $\mathbf{x}$  is a parametrization and find its image in  $M$ .

(a) *Ellipsoid*:

$$M : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

$$\mathbf{x}(u, v) = (a \cos u \cos v, b \cos u \sin v, c \sin u), \quad D : -\pi/2 < u < \pi/2.$$

(b) *Elliptic hyperboloid*:

$$M : \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1,$$

$$\mathbf{x}(u, v) = (a \cosh u \cos v, b \cosh u \sin v, c \sinh u), \quad \text{on } \mathbb{E}^3.$$

(c) *Elliptic hyperboloid* (two sheets):

$$M : \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1,$$

$$\mathbf{x}(u, v) = (a \sinh u \cos v, b \sinh u \sin v, c \cosh u), \quad D : u \neq 0.$$

#### Exercise 2.7

*Elliptic paraboloid*,  $M : z = (x^2/a^2) + (y^2/b^2)$ .

(a) Show that  $M$  is a surface and that

$$\mathbf{x}(u, v) = (au \cos v, bu \sin v, u^2), \quad u > 0,$$



is a parametrization which omits only one point of  $M$ .

(b) Describe the parameter curves of  $\mathbf{x}$  in general.

### Exercise 2.8

Hyperbolic paraboloid,  $M : z = (x^2/a^2) - (y^2/b^2)$ .

(a) Show that  $\mathbf{x} \mathbf{E}^3 \rightarrow \mathbf{E}^3$  is a proper patch covering all of  $M$ , where

$$\mathbf{x}(u, v) = (a(u+v), b(u-v), 4uv).$$

(b) Show that  $M$  is a doubly ruled surface by rewriting  $\mathbf{x}$  in ruled form in two different ways.

(c) Describe the parameter curves of  $\mathbf{x}$  in general.

## Differential Forms on a Surface

### Exercise 4.1

If  $\phi$  and  $\psi$  are 1-forms on a surface, prove that  $\phi \wedge \psi = -\psi \wedge \phi$ . Deduce that  $\phi \wedge \phi = 0$ .

## Topological Properties

### Exercise 6.1

Please ignore the reference to an exercise of *O'Neill*. (The reference has moved and does not contribute to solving the exercise.)

## 5 Part V

## Computational Techniques

The reading section is *O'Neill* sections 5.4 and 5.5. the exercises have been renumbered, so they are shown below in full.

### Exercise 4.1

For a Monge patch  $\mathbf{x}(u, v) = (u, v, f(u, v))$ , show that

$$E = 1 + f_u^2, \quad l = f_{uu}/W,$$

$$F = f_u f_v, \quad m = f_{uv}/W,$$

$$G = 1 + f_v^2, \quad n = f_{vv}/W,$$

where

$$W = (1 + f_u^2 + f_v^2)^{1/2}.$$

### Exercise 4.2

(Continuation) deduce that the image of  $\mathbf{x}$  is flat if and only if

$$f_{uu}f_{vv} - f_{uv}^2 = 0;$$

minimal if and only if

$$(1 + f_u^2)f_{vv} + (1 + f_v^2)f_{uu} - 2f_u f_v f_{uv} = 0.$$

### Exercise 4.3

Show that the image of the patch

$$\mathbf{x}(u, v) = (u, v, \log \cos v - \log \cos u)$$

is a minimal surface with Gaussian curvature

$$K = \frac{-\sec^2 u \sec^2 v}{W^4},$$

where

$$W^2 = 1 + \tan^2 u + \tan^2 v.$$

## 6 Part VI

### Orthogonal Coordinates

#### Exercise 3.2

(a) The page reference is to page 284.

## 7 Assignment Errata

### TMA02

Question 4: the formula for the curvature should be

$$\frac{1}{1 + k^2}$$

### TMA03

Question 3: Please disregard part (vi) asking about asymptotic directions. The 3 marks for this part will be awarded (1 mark each) to parts (iii)–(v)

### TMA04

Question 2(b): the third term of third entry should be  $f_{vv}\alpha_2'^2$  not  $f_v\alpha_2'^2$  as printed.

Exercise 2.4  
 (a) Show that the parametric curves of  $x$  in  $\mathbb{R}^3$  are...

### Exercise 2.4

Consider the paraboloid  $M = \{(x, y, z) \in \mathbb{R}^3 : z = x^2 + y^2\}$ .

(a) Show that  $x \in M \iff x = (u, v, u^2 + v^2)$  for some  $u, v \in \mathbb{R}$ .

$$x(u, v) = (u, v, u^2 + v^2)$$

(b) Show that  $M$  is a doubly ruled surface by exhibiting a family of straight lines in  $M$  that cover  $M$  in two different ways.

(c) Describe the parametric curves of  $x$  in  $M$  geometrically.

## Differential Forms on a Surface

### Exercise 3.1

Let  $\alpha$  and  $\beta$  be 1-forms on a surface  $M$  such that  $\alpha \wedge \beta = 0$  and  $\alpha \wedge d\alpha = 0$ . Show that  $\alpha = 0$  or  $\beta = 0$ .

## Topological Properties

### Exercise 4.1

Prove that the sphere  $S^2$  is not a manifold with boundary. (Hint: Consider the boundary of a disk.)

## 5 Part V

## Computational Techniques

Question 5: Please disregard part (ii) which is about programming. This question is for the part which will be awarded 1 mark each to parts (i)-(v).

### Exercise 5.1

Let  $x = (x_1, x_2, x_3) \in \mathbb{R}^3$  and  $y = (y_1, y_2, y_3) \in \mathbb{R}^3$ . Show that

$$x \wedge y = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1)$$

$$x \wedge x = 0$$

$$x \wedge (y \wedge z) = (x \wedge y) \wedge z + (x \wedge z) \wedge y$$

where

$$x \wedge y = (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1)$$

### Exercise 5.2

Let  $\alpha = x_1 dx_2 - x_2 dx_1$  and  $\beta = x_2 dx_3 - x_3 dx_2$ . Show that  $\alpha \wedge \beta = 0$ .

$$\alpha \wedge \beta = 0$$

where  $\alpha = x_1 dx_2 - x_2 dx_1$

$$(x_1 dx_2 - x_2 dx_1) \wedge (x_2 dx_3 - x_3 dx_2) = 0$$

### Exercise 5.3

Show that the form  $\alpha = x_1 dx_2 - x_2 dx_1$  is closed.



**STOP PRESS**

## **M434 SP3 2001**

**STOP PRESS 3 2001**

## **M434 DIFFERENTIAL GEOMETRY**

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## 1. The M500 Society/Revision Weekend

The 27th annual M500 Study/Revision Week-end, organized by the M500 Society, will run from Friday evening to Sunday afternoon, 14th-16th September 2001, at Aston University, Birmingham. Most maths-related courses are covered, including this one, and tuition is given by OU staff. Classes for this course run throughout the week-end, covering all the topics you need for the exam. It's a splendid opportunity to consolidate your coursework, to get started on your revision, and to meet other students.

Estimated prices for tuition and full board (two nights) are £130 en-suite, and £110 standard. There are 450 en-suite places and 300 standard places for residents. For those within commuting distance, there are non-residential, tuition-only places (no meals provided) at £55. There is a discount of £10 on those prices for current members of the M500 Society.

Bookings open at the end of March, and close on Monday 27th August. The event is likely to be over-subscribed, so please book early to avoid disappointment. For full details and a booking form, send a stamped, addressed envelope to:

Jeremy Humphries  
M500 Week-end 2001  
36 Penmanor  
Finstall  
Bromsgrove B60 3BZ.

Or visit the website at

<http://freespace.virgin.net/jeremy.humphries/sept.htm>

Please mention your course in all enquiries, as I may have specific information to send you.

The M500 Society is a mathematical society for OU students. Annual membership is £10. For details and an application form to join, send an SAE to:

Glenda Franklin  
M500  
16 Warbank Close  
Alvechurch  
Birmingham B48 7PA.

## 2. Past Examination Papers

**OUSA (Services) Ltd.** continues to offer a comprehensive range of **past examination papers** for Mathematics and Computing courses. For most courses a set of the previous four years papers are on sale (but only if the course has been in presentation for that number of years). Papers cost £1.00 each plus 50p postage and packing (minimum order £2.00)

In addition to this **OUSA (Services) Ltd.** also offers a wide range of **faculty specific products**. The range for the Faculty of Mathematics and Computing includes:-

T-shirt (XL only)	£7.50	Clipboard Folder	£5.30
OU Sweat shirt (Medium, XL)	£18.50	M206 T-shirt (XL only)	£10.00
Mug	£3.50	Postcard	20p
Mouse mat	£4.30		
UK Postage & Packing	£3.00		

Student membership cards, offering discounts over a range of products/services, can be obtained by sending a passport-sized photograph and student details to the address given below.

For an order form and full details of these products and other faculty ranges available, or past examination papers please telephone 01908 653693 (24hr ansaphone) or write to OUSA (Services) Ltd., PO Box 397, Walton Hall, Milton Keynes, MK7 6BE or Fax 01908 654326.

### 3. Postgraduate Courses in Computing

Why not continue your OU studies in computing at postgraduate level? The OU provides postgraduate level, professional development courses in computing, in its successful Computing for Commerce and Industry programme. The programme is recognised as being very effective for staff development – 60% of students have all or part of their course fees paid by their employers.

The courses are also recognised for CPD by the British Computer Society and the IEE

Course offered include

- software engineering
- architectures of computing systems
- project management
- user interface design and development
- object-oriented software development
- software development for networked applications using Java.
- relational database systems
- distributed applications and e-commerce

Courses have credit ratings of 15 or 30 points, and successful completion of 120 points of study results in the award of a Postgraduate Diploma. (You can also select courses from the Manufacturing: Management and Technology programme, or from the OU Business School to contribute towards a Diploma.) Once you have obtained the Diploma, you are eligible to register for the Project and dissertation module, which leads to a Master of Science degree.

You can obtain further information about the programme, and the related programme of manufacturing courses, by contacting the Call Centre, PO box 222, The Open University, Walton Hall, Milton Keynes MK7 6YY (Telephone 01908 653231) or viewing the CCI web site at <http://cci.open.ac.uk>

### 4. Stanley Collings Memorial Prize 2001

Stanley Collings took great delight in problems with a novel twist to them. Never mundane, those he composed were always intriguing and – above all – fun to solve. In his honour, we now invite you to submit a problem (or a suite of related problems) of your own which, whilst not necessarily wholly original in content, should be presented in an interesting setting. The problems will be judged according to all of the following criteria:

- attractiveness and interest in the way in which the problem is posed; the judges particularly stress the need for the problem to be presented in a pleasing manner;
- suitability of the problem for its potential solvers;
- clear explanation of mathematical concepts inherent in the problem and of the thinking processes involved in its solutions;
- development of the idea behind the problem and some suggestion as to how it might be generalised.

The winning entry will be awarded a prize of £100 and its composer will be notified early in 2002.

Participants should note the following regulations.

- Entry is restricted to those studying an Open University mathematics course (i.e. a course with an M in its code) in 2001.
- All entries should be accompanied by an Entry Form (see below).
- Entries should indicate who the problem is intended for (e.g. school children year 7-8, MST121 students, etc).
- Participants should provide a solution or an indication of a method of solution, where appropriate.



- Entries are expected to be no more than about four sides of A4 in length, and should be legible.
- Entries will become property of the Open University and may be posted on the Web
- The judges' decision will be final.

Please send your problem, together with this Entry Form, by 31st December 2001, to *Stanley Collings Memorial Prize, Courses Office, Mathematics and Computing Faculty, The Open University, Walton Hall, Milton Keynes MK7 6AA*.

# **STANLEY COLLINGS MEMORIAL PRIZE 2001 – ENTRY FORM**

Please write in capitals

Full name \_\_\_\_\_

Student number \_\_\_\_\_ Current course(s) \_\_\_\_\_

Address \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_ Daytime telephone number \_\_\_\_\_

Title of problem \_\_\_\_\_

Age and experience of intended solvers \_\_\_\_\_

I wish/do not wish\* my e-mail address to accompany my entry if it is posted on the Web: e-mail address: \_\_\_\_\_

*\*please delete as necessary*

M434 STOP PRESS 3 2001

Differential Geometry

**TMA02 Q3**

I am grateful to those who have pointed out an error in TMA02 Q3 and apologise for its presence.

Please note that the definition of  $B_\beta(0)$  should be

$$B_\beta(0) = (0, -1, 0).$$

(The question had  $(0, 1, 0)$  instead.)

As was pointed out to me, the resulting frame is left-handed, so cannot be a Frenet frame.

Bob Margolis

M434 STOP PRESS 4 2001

Differential Geometry

**TMA02 Q4**

I am grateful to those who have pointed out an error in TMA02 Q4 and apologise for its presence.

The expression for the curvature should read

$$\kappa_k = \frac{1}{1 + k^2}$$

Bob Margolis